Finite regular semigroups which are amalgamation bases for finite semigroups (Algebraic Semigroups, Formal Languages and Computation)

Shoji, Kunitaka

数理解析研究所講究録 1222 巻 2001 年 83-85

Finite regular semigroups which are amalgamation bases for finite semigroups

島根大学総合理工学部　庄司 邦孝 (Kunitaka Shoji)
Department of Mathematics,
Shimane University

Let $\mathcal{A}$ be a class of semigroups. A triple of semigroups S, T, U with $U = S \cap T$ being a subsemigroup of S and T is called an *amalgam* of S and T with a *core* U in $\mathcal{A}$ and denoted by $[S, T; U]$. An amalgama $[S, T; U]$ of $\mathcal{A}$ is *weakly embedable* in $\mathcal{A}$ if there exist a semigroup K belonging to $\mathcal{A}$ and monomorphisms $\xi_1 : S \to K$, $\xi_1 : T \to K$ such that the restrictions to U of ξ_1 and ξ_2 are equal to each other (that is, $\xi_1(S) \cap \xi_2(T) \supseteq \xi_1(U)$). In the case that $\xi_1(S) \cap \xi_2(T) = \xi_1(U)$, we say that an amalgama $[S, T; U]$ of $\mathcal{A}$ is *strongly embeddable* in $\mathcal{A}$. A semigroup U in $\mathcal{A}$ is *amalgamation base* [resp. *weak amalgamation base*] if any amalgam with a core U in $\mathcal{A}$ is strongly embeddable [resp. weakly embeddable] in $\mathcal{A}$. In this paper, we restrict ourselves to the cases that $\mathcal{A}$ is the class of all semigroups or the class of all finite semigroups. We will use the terms "*amalgamation base for semigroups*" or "*weak amalgamation base for finite semigroups*" in the former case or the latter.

Okniński and Putcha [8] proved that any finite semigroup U is an amalgamation base for all finite semigroups if the $\mathcal{J}$-classes of U is linearly ordered and the semigroup algebra $\mathbb{C}[U]$ over $\mathbb{C}$ has a zero Jacobson radical. As a by-products they obtained that any finite inverse semigroup U is an amalgamation base for all finite semigroups. In the paper [9] we gave a proof of the result by using representations of semigroups only. The purpose of this paper is to show that the same method enable to extend the the reslut from inverse semigroup to regular semigroups whose Rees factors satisfy the the conditions, Ann_l and Ann_r.

Let U be a semigroup with zero, 0, and $a, b \in S$.

The set $\{s \in U \mid sa = 0\}$ is called the *left annihilator* of a in S and is denoted by $ann_l(a)$.

In this case, we say that U satisfies the condition Ann_l if $ann_l(a) = ann_l(b)$ implies $aU = bU$.

The *right annihilator* and the condition Ann_r are defined by left-right duality.

Theorem 1. *Let U be a finite regular semigroup with a chain of ideals such that U_n is a maximal subgroup and each U_i/U_{i+1} is a completely 0-simple semigroups satisfying the conditions Ann_l and Ann_r $(1 \leq i \leq n-1)$. Then U is an amalgamation base for finite semigorups.*

Now we can present an example of a semigroup which does not satisfy the right annihilator condition but is an amalgamation base for finite semigroups

Let S_n be the symmetric group of degree n on the set $X = \{1, 2, \cdots, n\}$ and R_n the right zero semigroup X with multiplication $ij = j$ $(i, j \in X)$.
Let $U = S_n \cup R_n$ with multiplication of U :
$x \cdot \alpha =$ the image of x by α and $\alpha \cdot x = x$ ($x \in R_n$ and $\alpha \in S_n$).
Then U has the representation extension property and the free representation extension property. Consequently, U is an amalgamation base for semigroups (see [3]).

Theorem 2. *The semigroup $U = S_n \cup R_n$ with multiplication of U :
$x \cdot \alpha =$ the image of x by α and $\alpha \cdot x = x$ ($x \in R_n$ and $\alpha \in S_n$). U is an amalgamation base for finite semigroups or not.*

References

[1] S. Bulman-Fleming and K. McDowell, *Absolutely flat semigroups.* Pacific J. Math. **107**(1983), 319-333.

[2] A. H. Clifford and G. B. Preston, *Algebraic theory of semigroups*, Amer. Math. Soc., Math. Survey, No.7, Providence, R.I., Vol.I(1961); Vol.II(1967).

[3] T. E. Hall. *Representation extension and amalgamation for semigroups.* Quart. J. Math. Oxford (2) **29**(1978), 309-334.

[4] T. E. Hall. *Finite inverse semigroups and amalgamations.* Semigroups and their applications, Reidel, 1987, pp. 51-56.

[5] T. E. Hall and M.S. Putcha, *The potential $\mathcal{J}$-relation and amalgamation bases for finite semigroups*, Proc. Amer. Math. Soc. **95**(1985), 309-334.

[6] T. E. Hall and K. Shoji, *Finite bands and amalgamation bases for finite semigroups*, Communications in algebras, To appear.

[7] B. H. Neumann, *An essay on free products of groups with amalgamations* Trans. Roy. Soc. London Ser. A **246**(1954), 503-554.

[8] J. Okniński and M.S. Putcha, *Embedding finite semigroup amalgams*, J. Austral. Math. Soc. (Series A) **51**(1991), 489-496.

[9] K. Shoji, *A proof of Okniński and Putcha's theorem*, Sumitted.